

2005 Mathematics
Higher
Finalised Marking Instructions

**These Marking Instructions have been prepared by Examination Teams
for use by SQA Appointed Markers when marking External Course
Assessments.**

Mathematics Higher

Instructions to Markers

1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.
2. Award one mark for each ‘bullet’ point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.
3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of marks(s) should be made.

This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (✓). This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (✗ or ✗✓). In appropriate cases attention may be directed to work which is not quite correct (eg bad form) but which has not been penalised, by underlining with a dotted or wavy line.

Work which is correct but inadequate to score any marks should be corrected with a double cross tick (✗✗).

5.
 - The total mark for each section of a question should be entered in red in the **outer** right hand margin, opposite the end of the working concerned.
 - Only the mark should be written, **not** a fraction of the possible marks.
 - These marks should correspond to those on the question paper and these instructions.
6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked.

Where a candidate has scored zero marks for any question attempted, “0” should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking instructions.

8. Do not penalise:
 - working subsequent to a correct answer
 - legitimate variations in numerical answers
 - correct working in the “wrong” part of the question
 - omission of units
 - bad form
9. No piece of work should be scored through without careful checking – even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme – answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for the mark(s).
10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.
11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a referral to the PA. Please see the general instructions for PA referrals.
12. No marks should be deducted at this stage for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.
13. Transcription errors: In general, as a consequence of a transcription error, candidates lose the opportunity of gaining either the first ic mark or the first pd mark.
14. Casual errors: In general, as a consequence of a casual error, candidates lose the opportunity of gaining either the first ic mark or the first pd mark.
15. **Do not write any comments on the scripts.** A revised summary of acceptable notation is given on page 4.

Summary

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

1. **Tick** correct working.
2. Put a mark in the **right-hand margin to match the marks allocations on the question paper.**
3. **Do not** write marks as fractions.
4. Put each mark **at the end** of the candidate’s response to the question.
5. **Follow through** errors to see if candidates can score marks subsequent to the error.
6. **Do not** write any comments on the scripts.

Higher Mathematics: A Guide to Standard Signs and Abbreviations

Remember – No comments on the scripts. Please use the following and nothing else.

Signs

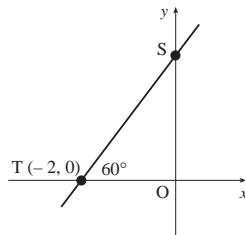
 The tick. You are not expected to tick every line but of course you must check through the whole of a response.	 The cross and underline. Underline an error and place a cross at the end of the line.	 The tick-cross. Use this to show correct work where you are following through subsequent to an error.	 The roof. Use this to show something is missing such as a crucial step in a proof of a ‘condition’ etc.	 The tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).	 The double-cross tick. Use this to show correct work but which is inadequate to score any marks. This may happen when working has been eased.	Bullets showing where marks have been allotted may be shown on scripts margins
				$\frac{dy}{dx} = 4x - 7$ $4x - 7 = \underline{0}$ $x = \frac{7}{4}$ $y = 3\frac{7}{8}$	 	2
				$C = \underline{(1, -1)}$ $m = \frac{3 - (-1)}{4 - 1}$ $m_{rad} = \frac{4}{3}$ $m_{tgt} = \frac{-1}{\frac{4}{3}}$ $m_{tgt} = -\frac{3}{4}$ $y - 3 = -\frac{3}{4}(x - 2)$		3
				$x^2 - 3x = 28$ $x = 7$	 	1
				$\sin(x) = 0 \cdot 75 = \text{inv} \sin(0 \cdot 75) = 48 \cdot 6^\circ$		1

**Remember – No comments on the scripts. No abbreviations. No new signs.
Please use the above and nothing else.**

All of these are to help us be more consistent and accurate.

Note: There is no such thing as a transcription error, a trivial error, a casual error or an insignificant error. These are all mistakes and as a consequence a mark is lost.

1 Find the equation of the line ST, where T is the point $(-2, 0)$ and angle STO is 60° .



3

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
1		3	C	G2, G3	NC	05/6

The primary method m/s is based on the following generic m/s.
THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE
GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE
THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD
SHOWN IN DETAIL IN THE MARKING SCHEME

- ¹ ss use $m = \tan \theta$
- ² pd use exact value
- ³ ic interpret result

Primary Method : Give 1 mark for each •

- ¹ $m = \tan(60^\circ)$ stated or implied by •²
- ² $m = \sqrt{3}$
- ³ $y - 0 = \sqrt{3}(x - (-2))$

3 marks

Notes

1 A candidate who states $m = \tan(\theta^\circ)$, and does not go on to use it earns no marks.

Incompletion 1

$$\begin{aligned}
 m &= \tan(60^\circ) \\
 y - 0 &= \tan(60^\circ)(x - (-2)) \\
 \bullet^1 &\times \checkmark \\
 \bullet^2 &\times \\
 \bullet^3 &\times \checkmark
 \end{aligned}$$

award 2 marks

Alternative Method 1

$$\begin{aligned}
 \bullet^1 \quad OS &= 2 \tan(60^\circ) = 2\sqrt{3} \\
 \bullet^2 \quad m &= \frac{2\sqrt{3}}{2} = \sqrt{3} \\
 &\quad (cf \ y = mx + c) \\
 \bullet^3 \quad y &= \sqrt{3}x + 2\sqrt{3}
 \end{aligned}$$

Alternative Method 2

$$\begin{aligned}
 \bullet^1 \quad \cos(60^\circ) &= \frac{2}{ST} \text{ leading to} \\
 ST &= 4 \text{ and } OS = \sqrt{12} \\
 \bullet^2 \quad m &= \frac{\sqrt{12}}{2} \\
 \bullet^3 \quad y - 0 &= \frac{\sqrt{12}}{2}(x - (-2))
 \end{aligned}$$

Common Error 1

$$\begin{aligned}
 m &= \sin(60^\circ) \\
 y - 0 &= \frac{\sqrt{3}}{2}(x - (-2)) \\
 \bullet^1 &\times \\
 \bullet^2 &\times \checkmark \\
 \bullet^3 &\times \checkmark
 \end{aligned}$$

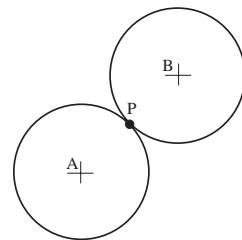
award 2 marks

2 Two congruent circles, with centres A and B, touch at P.

Relative to suitable axes, their equations are

$$x^2 + y^2 + 6x + 4y - 12 = 0 \text{ and } x^2 + y^2 - 6x - 12y + 20 = 0.$$

(a) Find the coordinates of P.
(b) Find the length of AB.



3
2

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
2	a	3	C	G9, G6	CN	05/18
	b	2	C	G9	CN	

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- ¹ ic interpret equ. of circle
- ² ic interpret equ. of circle
- ³ pd process midpoint
- ⁴ ss know how to find length
- ⁵ pd process

Primary Method : Give 1 mark for each •

- ¹ centre A = $(-3, -2)$
- ² centre B = $(3, 6)$
- ³ P = $(0, 2)$
- ⁴ $AB^2 = (3 - (-3))^2 + (6 - (-2))^2$ [CE 1]
- ⁵ AB = 10 [Note 2]

3 marks
2 marks

Notes

1 at •1, •2

Each of the following may be awarded 1 mark from the first two marks

$$A = (6, 4) \text{ and } B = (-6, -12)$$

$$A = (-6, -4) \text{ and } B = (6, 12)$$

$$A = (3, 2) \text{ and } B = (-3, -6)$$

2 At •5 stage, some errors lead to unsimplified surds.

DO NOT accept unsimplified square roots of perfect squares (up to 100).

e.g. $\sqrt{100}$ would not gain •5.

Alternative Method 1 for marks 1,2,3

$$p = \frac{1}{2}(b + a)$$

- ¹ $b = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$
- ² $a = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$
- ³ P = $(0, 2)$ [Note 1]

Notes

1 Treat P = $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ as bad form.

Alternative Method 2 for marks 4,5

- ⁴ $r^2 = 3^2 + 2^2 - (-12)$
or $r^2 = (-3)^2 + (-6)^2 - 20$
- ⁵ AB = $2r = 10$

Alternative Method 3 for marks 4,5

- ⁴ $\overrightarrow{AB} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$
- ⁵ AB = 10

Common Error 1 for (b)

$$AB^2 = (3 + (-3))^2 + (6 + (-2))^2$$

$$AB = 4$$

•⁴ \times

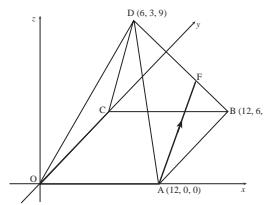
•⁵ $\times \checkmark$

award 1 mark for (b)

3 D,OABC is a pyramid. A is the point (12, 0, 0), B is (12, 6, 0) and D is (6, 3, 9). F divides DB in the ratio 2 : 1.

(a) Find the coordinates of the point F.

(b) Express \overrightarrow{AF} in component form.

4
1

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
3	a	4	C	G25	CN	05/24
	b	1	C	G17	CN	

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- ¹ ss know to find \overrightarrow{DB}
- ² ic interpret ratio
- ³ pd process scalar times vector
- ⁴ ic interpret vector and end points
- ⁵ ic interpret coordinates to vector

Primary Method : Give 1 mark for each •

$$\bullet^1 \overrightarrow{DB} = \begin{pmatrix} 12 - 6 \\ 6 - 3 \\ 0 - 9 \end{pmatrix}$$

$$\bullet^2 \overrightarrow{DF} = \frac{2}{3} \overrightarrow{DB}$$

$$\bullet^3 \overrightarrow{DF} = \frac{2}{3} \begin{pmatrix} 6 \\ 3 \\ -9 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix}$$

$$\bullet^4 D = (6, 3, 9) \text{ so } F = (10, 5, 3) \quad [\text{Note 1}] \quad 4 \text{ marks}$$

$$\bullet^5 \overrightarrow{AF} = \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix} \quad 1 \text{ mark}$$

Notes

- Do not penalise candidates who write the coordinates of F as a column vector (treat as bad form).
- A correct answer to (a) with no working may be awarded one mark only.
- For guessing the coordinates of F, no marks should be awarded in (a).
1 mark is still available in (b) provided the guess in (a) is geographically compatible with the diagram
ie $0 \leq x \leq 12$
 $3 \leq y \leq 6$
 $0 \leq z \leq 9$
- In (a)
Where the ratio has been reversed (ie 1:2) leading to $F = (8, 4, 6)$ then 3 marks may be awarded ($\bullet^1, \bullet^3, \bullet^4$).
- In (b)
Accept $\overrightarrow{AF} = -2\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$ for \bullet^5 .

Alternative Method 1 [Marks 1-4]

$$\bullet^1 \overrightarrow{DF} = 2\overrightarrow{FB} \quad \text{s/i by } \bullet^2$$

$$\bullet^2 \mathbf{f} - \mathbf{d} = 2\mathbf{b} - 2\mathbf{f}$$

$$\bullet^3 3\mathbf{f} = 2 \begin{pmatrix} 12 \\ 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \\ 9 \end{pmatrix}$$

$$\bullet^4 F = (10, 5, 3) \quad [\text{Note 1}]$$

Alternative Method 3 [Marks 1-5]

$$\bullet^1 \overrightarrow{AF} = \overrightarrow{AB} + \overrightarrow{BF}$$

$$\bullet^2 \overrightarrow{AF} = \overrightarrow{AB} + \frac{1}{3} \overrightarrow{BD}$$

$$\bullet^3 \overrightarrow{AF} = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 6 \\ 3 \\ 9 \end{pmatrix} - \begin{pmatrix} 12 \\ 6 \\ 0 \end{pmatrix}$$

$$\bullet^4 \overrightarrow{AF} = \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$$

$$\bullet^5 (A = (12, 0, 0) \text{ so } F = (10, 5, 3))$$

Alternative Method 2 [Marks 1-4]

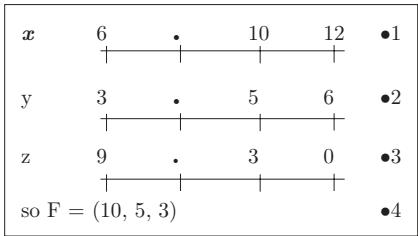
$$\bullet^1 \mathbf{f} = \frac{m\mathbf{b} + n\mathbf{d}}{m + n} \quad \text{s/i by } \bullet^3$$

$$\bullet^2 m = 2, n = 1 \quad \text{s/i by } \bullet^3$$

$$\bullet^3 \mathbf{f} = \frac{1}{3} \left(2 \begin{pmatrix} 12 \\ 6 \\ 0 \end{pmatrix} + 1 \begin{pmatrix} 6 \\ 3 \\ 9 \end{pmatrix} \right)$$

$$\bullet^4 F = (10, 5, 3) \quad [\text{Note 1}]$$

Alternative Method 4 [Marks 1-4]



4 Functions $f(x) = 3x - 1$ and $g(x) = x^2 + 7$ are defined on the set of real numbers.

(a) Find $h(x)$ where $h(x) = g(f(x))$. 2

(b) (i) Write down the coordinates of the minimum turning point of $y = h(x)$. 2

(ii) Hence state the range of the function h . 2

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
4	a	2	C	A4	NC	05/7
	b	2	C	A1	NC	

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- ¹ ic interpret comp. function build-up
- ² ic interpret comp. function build-up
- ³ ic interpret function
- ⁴ ic interpret function

Primary Method : Give 1 mark for each •

- ¹ $g(3x - 1)$ *stated or implied by •2*
- ² $(3x - 1)^2 + 7$ 2 marks
- ³ $\left(\frac{1}{3}, 7\right)$ [Note 1]
- ⁴ $y \geq 7$ [Note 2] 2 marks

Notes

1 For •3

No justification is required for •3. Candidates may choose to differentiate etc but may still only earn one mark for a correct answer.

2 For •4

Accept $y > 7$, $h \geq 7$, $h > 7$, $h(x) > 7$, $h(x) \geq 7$

Do not accept $x \geq 7$, $x > 7$

Common Error No.1

- ¹ \times $f(x^2 + 7)$
- ² $\times \checkmark$ $3x^2 + 20$
- ³ $\times \checkmark$ $(0, 20)$
- ⁴ $\times \checkmark$ $y \geq 20$

award 3 marks

Notes 1 & 2 apply.

5 Differentiate $(1 + 2 \sin(x))^4$ with respect to x .

2

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
5		2	A	C20, C21	CN	05/28

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- ¹ pd start differentiation process
- ² pd use the chain rule

Primary Method : Give 1 mark for each •

- ¹ $4(1 + 2 \sin(x))^3$
- ² ... $\times 2 \cos(x)$

2 marks

Common Error 1

- ¹ $\times 1 + 2 \sin^4(x)$
- ² $\times \sqrt{8 \sin^3(x) \times \cos(x)}$

award 1 mark

Common Error 2

- ¹ $\times 1 + 16 \sin^4(x)$
- ² $\times \sqrt{64 \sin^3(x) \times \cos(x)}$

award 1 mark

Common Error 3

[mixture of differentiating and integrating]

- ¹ $\times \frac{1}{4}(1 + 2 \sin(x))^3$
- ² $\times \times \frac{1}{2} \cos(x)$

award 0 marks

Common Error 4

- ¹ $\times 4(1 + 2 \sin(x))^5$
- ² $\times \sqrt{\times 2 \cos(x)}$

award 1 mark

6 (a) The terms of a sequence satisfy $u_{n+1} = ku_n + 5$. Find the value of k which produces a sequence with a limit of 4. 2

(b) A sequence satisfies the recurrence relation $u_{n+1} = mu_n + 5$, $u_0 = 3$.

(i) Express u_1 and u_2 in terms of m .

(ii) Given that $u_2 = 7$, find the value of m which produces a sequence with no limit. 5

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
6	a	2	C	A13	CN	05/42
	b	5	B	A11, A13	CN	

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- ¹ ss know how to find limit
- ² pd process
- ³ ic interpret rec. relation
- ⁴ ic interpret rec. relation
- ⁵ pd arrange in standard form
- ⁶ pd process a quadratic
- ⁷ ic use limit condition

Primary Method : Give 1 mark for each •

- ¹ e.g. $4 = k \times 4 + 5$ [Notes 1,2,3]
- ² $k = -\frac{1}{4}$ 2 marks
- ³ $u_1 = 3m + 5$
- ⁴ $u_2 = m(3m + 5) + 5$ [Note 4]
 $(m(3m + 5) + 5 = 7)$
- ⁵ $3m^2 + 5m - 2 = 0$ [Note 5]
- ⁶ $(3m - 1)(m + 2) = 0$
- ⁷ $m = -2$ 5 marks

Alternative Method 1 for (a)

$$\text{Using } L = \frac{b}{1-a}$$

$$\bullet^1 \quad 4 = \frac{5}{1-k}$$

$$\bullet^2 \quad k = -\frac{1}{4}$$

Alternative Method 2 for (a)

$$L = kL + 5$$

$$kL = L - 5$$

$$\bullet^1 \quad k = \frac{L-5}{L}$$

$$\bullet^2 \quad k = \frac{4-5}{4} = -\frac{1}{4}$$

Notes for (a)

1 Guess and Check

Guessing $k = -0.25$ and checking algebraically or iteratively that this does yield a limit of 4 may be awarded 1 mark.

2 No working

Simply stating that $k = -0.25$ earns no marks.

3 Wrong formula

Work using an incorrect 'formula' leading to a valid value of k (ie $|k| < 1$) may be awarded 1 mark.

for (b)

4 If u_2 is not a quadratic, then no further marks are available.

5 An “=0” must appear at least once in working at the •5/6 stage.

6 For candidates who make errors leading to no values outside the range $-1 < m < 1$, or to two values outside the range, then they must say why they are accepting or rejecting in order to gain •7

7 For •7, either crossing out the “1/3” or underlining the “-2” is the absolute minimum communication required for this i/c mark. [A statement would be preferable]

Common Error 1

- ¹ $\times 4 = \frac{5}{1-a}$
- ² $\times \sqrt{a} = -\frac{1}{4}$

award 1 mark

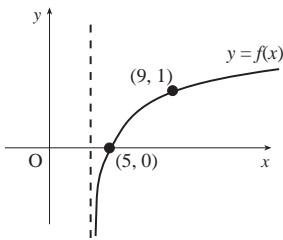
Common Error 2

- ³ $\checkmark u_1 = 3m + 5$
- ⁴ $\times u_2 = 3m^2 + 5$
- ⁵ $\times 3m^2 = 2$ or equivalent
- ⁶ $\times m = \sqrt{\frac{2}{3}}$ (eased)
- ⁷ $\times \checkmark$ there are no values which do not yield a limit award 2 marks

7 The function f is of the form $f(x) = \log_b(x - a)$.

The graph of $y = f(x)$ is shown in the diagram.

(a) Write down the values of a and b .
 (b) State the domain of f .



2
1

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
7	a	2	C	A7	NC	05/9
	b	1	C	A1	NC	

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- ¹ ic interpret the translation
- ² ic interpret the base
- ³ ic interpret diagram

Primary Method : Give 1 mark for each •

<ul style="list-style-type: none"> •¹ $a = 4$ •² $b = 5$ •³ domain is $x > a$ 	<p>[Note 1]</p> <p>[Note 2]</p>	<p>2 marks</p> <p>1 mark</p>
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Notes

1 No justification is required for marks 1 and 2.
 BUT simply stating

$$0 = \log_b(5 - a) \text{ and } 1 = \log_b(9 - a)$$

with no further work earns no marks.

However

$$1 = \log_b(9 - a) \text{ and } b = 9 - a$$

may be awarded 1 mark.

Of course to gain the other mark, both values would need to be stated.

2 Clearly $x > 4$ is correct

but **do not** accept a domain of $x \geq 4$.

8 A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number.

- Show that $(x-3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully.
- Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x - and y -axes.
- Find the greatest and least values of f in the interval $-2 \leq x \leq 2$.

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
8	a	5	C	A21	NC	05/10
	b	2	C	A21	NC	
	c	5	B	C11	NC	

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- ¹ ss know to use $x = 3$
- ² pd complete strategy
- ³ ic interpret zero remainder
- ⁴ ic interpret quadratic factor
- ⁵ pd complete factorising

Notes

In the Primary method, (a)

- 1 Candidates must show some acknowledgement of the result of the synthetic division. Although a statement w.r.t. the zero is preferable, accept something as simple as "underlining" the zero.
- 2 Candidates may use a second synthetic division to complete the factorisation. •4 and •5 are available.

Primary Method : Give 1 mark for each •

- ¹ eg

3	2	-7	0	9
---	---	----	---	---
- ² eg

3	2	-7	0	9
	6	-3	-9	

$$\begin{array}{ccccc} & 2 & -1 & -3 & 0 \end{array}$$
- ³ remainder is zero so $(x - 3)$ is a factor **[Note 1]**
- ⁴ $2x^2 - x - 3$
- ⁵ $(x - 3)(2x - 3)(x + 1)$ **stated explicitly** 5 marks

Alternative method 1 (marks 1-5) (linear factor by substitution)

- ¹ $f(3) = \dots$
- ² $f(3) = 2 \times 3^3 - 7 \times 3^2 + 9 = 54 - 63 + 9 = 0$
- ³ eg 3

2	-7	0	9
6			

2	-1	-3	0
---	----	----	---
- ⁴ $2x^2 - x - 3$
- ⁵ $(x - 3)(2x - 3)(x + 1)$

Alternative method 3 (marks 1-5) (quad factor by inspection)

- ¹ $f(3) = \dots$
- ² $f(3) = 2 \times 3^3 - 7 \times 3^2 + 9 = 54 - 63 + 9 = 0$
- ³ $(x - 3)(2x^2 \dots)$
- ⁴ $(x - 3)(2x^2 - x - 3)$
- ⁵ $(x - 3)(2x - 3)(x + 1)$

Alternative method 2 (marks 1-5) (long division)

$$\begin{array}{r}
 \bullet^1 \quad x \quad -3 \quad \left| \begin{array}{r} 2x^2 \\ 2x^3 - 7x^2 \\ \hline 2x^3 - 6x^2 \\ \hline -x^2 \end{array} \right. \quad +9 \\
 \\
 x \quad -3 \quad \left| \begin{array}{r} 2x^2 \quad -x \quad -3 \\ 2x^3 - 7x^2 \quad \quad \quad +9 \\ \hline 2x^3 - 6x^2 \\ \hline -x^2 \end{array} \right. \\
 \\
 \bullet^2 \quad \left| \begin{array}{r} -x^2 \quad +3x \\ -3x \quad +9 \\ \hline -3x \quad +9 \end{array} \right.
 \end{array}$$

8 A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number.

(a) Show that $(x-3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully. 5

(b) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x - and y -axes. 2

(c) Find the greatest and least values of f in the interval $-2 \leq x \leq 2$. 5

Primary Method : Give 1 mark for each •		
• ⁶ ic interpret y -intercept		
• ⁷ ic interpret x -intercepts		
• ⁸ ss set derivative to zero		
• ⁹ pd solve		
• ¹⁰ ss evaluate function at an end point		
• ¹¹ ic interpret results		
• ¹² ic interpret results		
	[Note 3]	2 marks
• ⁸ $6x^2 - 14x = 0$		
• ⁹ $x = 0$ or $x = \frac{14}{6}$	[Note 6]	
• ¹⁰ $f(-2) = -35$ OR $f(2) = -3$		
• ¹¹ greatest value = 9		
• ¹² least value = -35	[Note 7]	5 marks

Notes

In the Primary method (b)

3 Only coordinates are acceptable for full marks. Simply stating the values at which it cuts the x - and y -axes may be awarded 1 mark (out of 2).

4 If all the coordinates are “round the wrong way” award 1 mark.

5 If the brackets are missing, treat as bad form.

In the Primary method (c)

6 Ignore any attempt to evaluate function at $x = 7/3$.

7 •11 and •12 are not available unless both end points and the st. points have been considered.

In the Alt.5 method (c)

8 •12 is not available unless both end points have been considered.

In (c)

9 Some candidates simply draw up a table using integer values from -2 to 2 and make conclusions from it. This earns •9 (Primary) ONLY, provided that one of the end points is correct.

Alternative method 5 (marks 8-12) (nature table)

• ⁸ $6x^2 - 14x = 0$		
• ⁹ $x = 0$ or $x = \frac{14}{6}$	[Note 6]	
• ¹⁰ nature table showing $x = 0$ is max. tp and the greatest (maximum) value is 9		
• ¹¹ $f(-2) = -35$ OR $f(2) = -3$		
• ¹² least value = -35	[Note 8]	

9 If $\cos(2x) = \frac{7}{25}$ and $0 < x < \frac{\pi}{2}$, find the exact values of $\cos(x)$ and $\sin(x)$.

4

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
9		4	C	T8	NC	05/16

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SHOWN IN DETAIL IN THE MARKING SCHEME

- ¹ ss use double angle formula
- ² pd process
- ³ pd process
- ⁴ pd process

Primary Method : Give 1 mark for each •

- ¹ $2\cos^2(x) - 1 = \frac{7}{25}$
- ² $\cos^2(x) = \frac{32}{50}$
- ³ $\cos(x) = \frac{4}{5}$
- ⁴ $\sin(x) = \frac{3}{5}$

4 marks

Notes

1 In the event of $\cos^2(x) - \sin^2(x)$ being used, no marks are available until the equation reduces to a quadratic in either $\cos(x)$ or $\sin(x)$.

2 $\cos(x) = \pm \frac{4}{5}$, $\sin(x) = \pm \frac{3}{5}$ loses •3.

3 •3 and •4 are only available as a consequence of attempting to apply the double angle formula.
(This note does not apply to alt. method 2)

4 Guess and Check.

For guessing that $\cos(x) = \frac{4}{5}$ and $\sin(x) = \frac{3}{5}$,

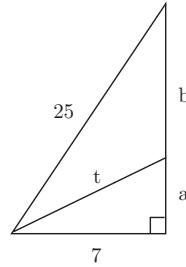
substituting them into any valid expression for $\cos(2x)$ and getting 7/25, award 1 mark only.

Alternative Method 1

- ¹ $1 - 2\sin^2(x) = \frac{7}{25}$
- ² $\sin^2(x) = \frac{18}{50}$
- ³ $\sin(x) = \frac{3}{5}$
- ⁴ $\cos(x) = \frac{4}{5}$

Alternative Method 2

- ¹ $(7, 24, 25)$ triangle $\Rightarrow a + b = 24$
and angle bisector $\Rightarrow \frac{a}{b} = \frac{7}{25}$
- ² $a + \frac{25}{7}a = 24 \Rightarrow a = \frac{21}{4}$
- ³ $(21, 28, 35)$ triangle $\Rightarrow t = \frac{35}{4}$
- ⁴ $\cos(x) = \frac{4}{5}$ **and** $\sin(x) = \frac{3}{5}$



Common Error 1

$2\cos^2(x) - 1 = \frac{7}{25}$
 $\cos^2(x) = \frac{64}{25}$
 $\cos(x) = \frac{8}{5}$
 $\sin(x) = \frac{6}{5}$
•¹ ✓ •² ✗, •³ ✗, •⁴ ✗
award 1 mark only

Common Incompletion 1

- ¹ ✓ $2\cos^2(x) - 1 = \frac{7}{25}$
- ² ✓ $\cos^2(x) = \frac{32}{50}$
- ³ ✗ $\cos(x) = \sqrt{\frac{32}{50}}$
- ⁴ ✗✓ $\sin(x) = \sqrt{\frac{18}{50}}$

award 3 marks

10 (a) Express $\sin(x) - \sqrt{3}\cos(x)$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \leq a \leq 2\pi$.

4

(b) Hence, or otherwise, sketch the curve with equation $y = 3 + \sin(x) - \sqrt{3}\cos(x)$ in the interval $0 \leq x \leq 2\pi$.

5

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
10	a	4	C	T13	NC	05/27
	b	5	A	T15	NC	

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- ¹ ic expand
- ² ic compare coefficients
- ³ pd process k
- ⁴ pd process angle
- ⁵ ic state equation
- ⁶ ic completing graph
- ⁷ ic completing graph
- ⁸ ic completing graph
- ⁹ ic completing graph

Notes

In the whole question

Do not penalise more than once for not using radians.

In (a)

- 1 $k(\sin(x)\cos(a) - \cos(x)\sin(a))$ is acceptable for •1
- 2 No justification is required for •3
- 3 •3 is not available for an unsimplified $\sqrt{4}$
- 4 $2(\sin(x)\cos(a) - \cos(x)\sin(a))$ or $2\sin(x)\cos(a) - 2\cos(x)\sin(a)$ is acceptable for •1 and •3
- 5 Candidates may use any form of the wave equation to start with as long as their final answer is in the form $k\sin(x - a)$. If it is not, then •4 is not available.
- 6 •4 is only available for an answer in radians.
- 7 Treat $k\sin(x)\cos(a) - \cos(x)\sin(a)$ as bad form only if •2 is gained.

In (b)

- 8 The correct sketch need not include annotation of max, min or intercept for •5 to be awarded but you would need to see the graph lying between $y = 1$ and $y = 5$.
- 9 •6 is available for one cycle of any sinusoidal curve of period 2π except $y = \sin(x)$. Some evidence of a scale is required.
- 10 For •7, accept 1.3 in lieu of $3 - \sqrt{3}$
- 11 Do not penalise graphs which go beyond the interval $0 \dots 2\pi$.

Primary Method : Give 1 mark for each •

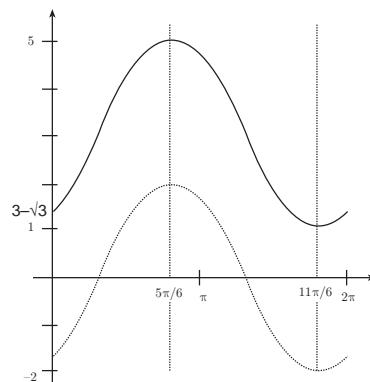
• ¹ $k\sin(x)\cos(a) - k\cos(x)\sin(a)$	STATED EXPLICITLY
• ² $k\cos(a) = 1, k\sin(a) = \sqrt{3}$	STATED EXPLICITLY
• ³ $k = 2$	[Notes 1-7]
• ⁴ $a = \frac{\pi}{3}$	4 marks
• ⁵ $y = 3 + 2\sin\left(x - \frac{\pi}{3}\right)$	stated or implied by a correct sketch [Note 8]
• ⁶ a sinusoidal curve	[Notes 9,10]
• ⁷ y -intercept at $(0, 3 - \sqrt{3})$ and no x -intercepts	5 marks
• ⁸ max at $\left(\frac{5\pi}{6}, 5\right)$	
• ⁹ min at $\left(\frac{11\pi}{6}, 1\right)$	

Alternative marking for •8 and •9

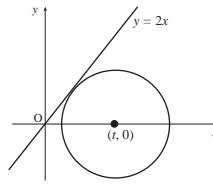
• ⁸ max at $x = \frac{5\pi}{6}$ and min at $x = \frac{11\pi}{6}$
• ⁹ graph lies between $y = 1$ and $y = 5$

Alternative method for •5 to •9 (Calculus)

• ⁵ $\frac{dy}{dx} = \cos(x) + \sqrt{3}\sin(x) = 0$
• ⁶ $\tan(x) = -\frac{1}{\sqrt{3}}$
• ⁷ max at $\left(\frac{5\pi}{6}, 5\right)$
• ⁸ min at $\left(\frac{11\pi}{6}, 1\right)$
• ⁹ $x = 0 \Rightarrow y = 3 - \sqrt{3}$ and annotated sketch.



11 (a) A circle has centre $(t, 0)$, $t > 0$, and radius 2 units.
Write down the equation of the circle.
(b) Find the exact value of t such that the line $y = 2x$ is a tangent to the circle.

1
5

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
11	a	1	C	G10	CN	05/28
	b	4	A	G13	CN	

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- ¹ ic state equ. of circle
- ² ss substitute
- ³ pd rearrange in standard form.
- ⁴ ss know to use "discriminant = 0"
- ⁵ ic identify "a", "b" and "c"
- ⁶ pd process

Notes

- 1 Subsequent to trying to use an expression masquerading as the discriminant e.g. $a^2 - 4bc = 0$, **only •5** (from the last two marks) is still available.
- 2 Treat $t = \pm\sqrt{5}$ as bad form.

Primary Method : Give 1 mark for each •

- ¹ $(x - t)^2 + (y - 0)^2 = 2^2$ **1 mark**
- ² $(x - t)^2 + (2x)^2 = 4$
- ³ $5x^2 - 2tx + t^2 - 4 = 0$
- ⁴ " $b^2 - 4ac = 0$ " **[Note 1]**
- ⁵ $a = 5, b = -2t, c = t^2 - 4$
- ⁶ $4t^2 - 20(t^2 - 4) = 0$
and $t = \sqrt{5}$ **[Note 2]** **5 marks**

Common Error No. 1

- ⁵ $\times a = 5, b = -2, c = t^2 - 4$
- ⁶ $4 - 20(t^2 - 4) = 0$
 $20t^2 = 84$
 $\times\sqrt{t} = \sqrt{\frac{21}{5}}$ or $\sqrt{4.2}$

Alternative Method 1 (for (b))

Let P be point of contact, C the centre of the circle.
Consider triangle OPC .

- $OPC = 90^\circ$ (tgt/radius)
- $PC = 2$ (radius)
- $CP/OP = \tan(COP) = 2$ (gradient of tgt)
- Hence $OP = 1$
- and, by Pythagoras, $t = OC = \sqrt{(2^2 + 1^2)} = \sqrt{5}$.

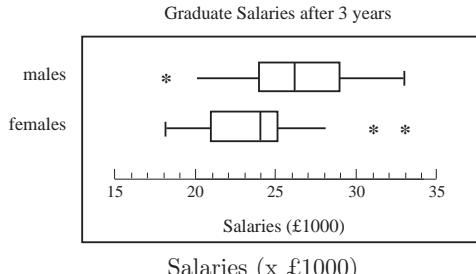
Alternative Method 2 (for (b))

$$y = 2x \Rightarrow m_{tgt} = 2 \text{ and } m_{rad} = -\frac{1}{2}$$

- ² equ of radius is $x + 2y = t$
ie $x - t = -2y$
- ³ $(-2y)^2 + y^2 = 4$
- ⁴ $y = \frac{2}{\sqrt{5}}$
- ⁵ $x = \frac{1}{2}y \Rightarrow x = \frac{1}{\sqrt{5}}$
- ⁶ $t = x + 2y \Rightarrow t = \sqrt{5}$

S1 The boxplot shows the salaries of male and female graduates working for a large company at the end of their third year of employment.

Compare the salaries of these males and females.



3

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
S1		3	C	4.1.3/4	NC	05/70

- ¹ ic comment
- ² ic comment
- ³ ic comment

- ¹ one comment from list
- ² one comment from list
- ³ one comment from list

3 marks

males had higher salaries on average by £2000

range of salaries is broadly similar

only 2 females achieved same salary as top 25% males

majority of males earned more than the average female

any other reasonable comment

S2 A bag contains 4 blue and 2 red counters. 2 counters are drawn at random without replacement.

The random variable X is the number of blue counters drawn.

[5] (a) Find the probability distribution for X . 4
 (b) Find $E(X)$. 2

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
S2		6	C	4.2.11/12	NC	05/22

- ¹ ss know to find $P(X=0)$ etc
- ² pd process
- ³ pd process
- ⁴ pd process
- ⁵ ss choose correct form
- ⁶ pd process

- ¹ $\frac{1}{15}$
- ² $\frac{2}{6} \times \frac{4}{5}$
- ³ $\dots + \frac{4}{6} \times \frac{2}{5} = \frac{8}{15}$
- ⁴ $\frac{2}{5}$
- ⁵ $E(X) = \Sigma xp(x)$
- ⁶ $\frac{4}{3}$

4 marks

2 marks

S3 A continuous random variable T has probability density function $f(t) = \begin{cases} 5t^k & 0 < t < 1 \\ 0 & \text{otherwise} \end{cases}$ [10]

(a) Find the value of k . (b) Calculate $P(0 < T < \frac{1}{2})$. 4,3

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
S3		6	B	4.3.2	CN	05/65

- ¹ ss know $\int_0^1 f(t)dt = 1$
- ² pd process
- ³ pd process
- ⁴ pd process
- ⁵ ss use $\int_0^1 5t^k dt$
- ⁶ pd integrate
- ⁷ pd process limits

- ¹ $\int_0^1 f(t)dt = 1$
- ² $\int_0^1 5t^k dt = \frac{5}{k+1} t^{k+1}$
- ³ $\frac{5}{k+1}$
- ⁴ $k = 4$
- ⁵ $\int_0^{\frac{1}{2}} 5t^4 dt$
- ⁶ $\left[t^5 \right]_0^{\frac{1}{2}}$
- ⁶ $\frac{1}{32}$

4 marks

3 marks

17

Mathematics Higher

Instructions to Markers

1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.
2. Award one mark for each ‘bullet’ point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.
3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of marks(s) should be made.

This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (✓). This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (✗ or ✗✓). In appropriate cases attention may be directed to work which is not quite correct (eg bad form) but which has not been penalised, by underlining with a dotted or wavy line.

Work which is correct but inadequate to score any marks should be corrected with a double cross tick (✗✗).

5.
 - The total mark for each section of a question should be entered in red in the **outer** right hand margin, opposite the end of the working concerned.
 - Only the mark should be written, **not** a fraction of the possible marks.
 - These marks should correspond to those on the question paper and these instructions.
6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked.

Where a candidate has scored zero marks for any question attempted, “0” should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking instructions.

8. Do not penalise:
 - working subsequent to a correct answer
 - legitimate variations in numerical answers
 - correct working in the “wrong” part of the question
 - omission of units
 - bad form
9. No piece of work should be scored through without careful checking – even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme – answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for the mark(s).
10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.
11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a referral to the PA. Please see the general instructions for PA referrals.
12. No marks should be deducted at this stage for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.
13. Transcription errors: In general, as a consequence of a transcription error, candidates lose the opportunity of gaining either the first ic mark or the first pd mark.
14. Casual errors: In general, as a consequence of a casual error, candidates lose the opportunity of gaining either the first ic mark or the first pd mark.
15. **Do not write any comments on the scripts.** A revised summary of acceptable notation is given on page 4.

Summary

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

1. **Tick** correct working.
2. Put a mark in the **right-hand margin to match the marks allocations on the question paper.**
3. **Do not** write marks as fractions.
4. Put each mark **at the end** of the candidate’s response to the question.
5. **Follow through** errors to see if candidates can score marks subsequent to the error.
6. **Do not** write any comments on the scripts.

Higher Mathematics: A Guide to Standard Signs and Abbreviations

Remember – No comments on the scripts. Please use the following and nothing else.

Signs

 The tick. You are not expected to tick every line but of course you must check through the whole of a response.	 The cross and underline. Underline an error and place a cross at the end of the line.	 The tick-cross. Use this to show correct work where you are following through subsequent to an error.	 The roof. Use this to show something is missing such as a crucial step in a proof of a ‘condition’ etc.	 The tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).	 The double-cross tick. Use this to show correct work but which is inadequate to score any marks. This may happen when working has been eased.	Bullets showing where marks have been allotted may be shown on scripts margins
				$\frac{dy}{dx} = 4x - 7$ $4x - 7 = \underline{0}$ $x = \frac{7}{4}$ $y = 3\frac{7}{8}$	 	2
				$C = \underline{(1, -1)}$ $m = \frac{3 - (-1)}{4 - 1}$ $m_{rad} = \frac{4}{3}$ $m_{tgt} = \frac{-1}{\frac{4}{3}}$ $m_{tgt} = -\frac{3}{4}$ $y - 3 = -\frac{3}{4}(x - 2)$		3
				$x^2 - 3x = 28$ $x = 7$	 	1
				$\sin(x) = 0 \cdot 75 = \text{inv} \sin(0 \cdot 75) = 48 \cdot 6^\circ$		1

**Remember – No comments on the scripts. No abbreviations. No new signs.
Please use the above and nothing else.**

All of these are to help us be more consistent and accurate.

Note: There is no such thing as a transcription error, a trivial error, a casual error or an insignificant error. These are all mistakes and as a consequence a mark is lost.

1 Find $\int \frac{4x^3 - 1}{x^2} dx, x \neq 0.$

4

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
1		4	C	C14, C13	CN	05/20

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- ¹ ss: arrange in integrable form
- ² pd: integrate positive index
- ³ pd: integrate negative index
- ⁴ ic: complete including const. of int.

Primary Method : Give 1 mark for each •

$$\bullet^1 4x - x^{-2}$$

$$\bullet^2 \frac{4x^2}{2}$$

$$\bullet^3 -\frac{x^{-1}}{-1}$$

$$\bullet^4 2x^2 + x^{-1} + c$$

[Note 3]

4 marks

Notes

- 1 If incorrectly expressed in integrable form, follow throughs must match the generic marking scheme.
- 2 •³ can only be awarded on follow through provided the integral involves a negative index.
- 3 •⁴ can only be awarded if the constant of integration appears somewhere in the working.
- 4 •⁴ can only be awarded as a result of at least one valid integration at the •² or •³ stage.

Common Error 1

- ¹ $\times 4x - 1$
- ² $\times \sqrt{2x^2}$
- ³ $\times -x$ [see Generic •³]
- ⁴ $\times \sqrt{2x^2 - x + c}$

award 2 marks

Common Error 2

- ¹ $\times 4x^3 - 1 - x^{-2}$
- ² $\times \sqrt{\frac{4x^4}{4} - x}$
- ³ $\times \sqrt{-\frac{x^{-1}}{-1}}$
- ⁴ $\times \sqrt{x^4 - x + x^{-1} + c}$

award 3 marks

Common Error 4

$$\frac{x^4 - x}{\frac{1}{2}x^2} + c$$

award 0 marks

Common Error 5

$$\begin{aligned} &(4x - 1)x^{-2} \\ &(2x^2 - x)(-x^{-1}) + c \end{aligned}$$

award 0 marks

Common Error 3

- ¹ $\times 4x^3 - 1 + x^{-2}$
- ² $\times \sqrt{\frac{4x^4}{4} - x}$
- ³ $\times \sqrt{\frac{x^{-1}}{-1}}$
- ⁴ $\times \sqrt{x^4 - x - x^{-1} + c}$

award 3 marks

Common Error 6

$$\begin{aligned} &(4x^3 - 1)x^{-2} \\ &(x^4 - x)(-x^{-1}) + c \end{aligned}$$

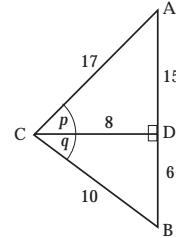
award 0 marks

**Throughout this paper, unless specifically mentioned,
a correct answer with no working receives no credit.**

2 Triangles ACD and BCD are right-angled at D with angles p and q and lengths as shown in the diagram.

(a) Show that the exact value of $\sin(p+q)$ is $\frac{84}{85}$.

(b) Calculate the exact values of
(i) $\cos(p+q)$ (ii) $\tan(p+q)$.

4
3

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
2	a	4	C	T9	CN	05/41
	b	3	C	T9	CN	

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- ¹ ic: interpret diagram
- ² ic: interpret diagram
- ³ ss: expand $\sin(A+B)$
- ⁴ pd: sub. and complete
- ⁵ ss: expand $\cos(A+B)$
- ⁶ pd: sub. and complete
- ⁷ ic: use $\tan(x) = \sin(x) / \cos(x)$

Primary Method : Give 1 mark for each •

- ¹ $\cos(p) = \frac{8}{17}, \sin(p) = \frac{15}{17}$ [Note 1]
stated or implied by •4 when written in the same order as •3
- ² $\cos(q) = \frac{8}{10}, \sin(q) = \frac{6}{10}$
- ³ $\sin(p)\cos(q) + \cos(p)\sin(q)$ explicitly stated
- ⁴ $\frac{15}{17} \times \frac{8}{10} + \frac{8}{17} \times \frac{6}{10} = \& \text{complete}$ 4 marks
- ⁵ $\cos(p)\cos(q) - \sin(p)\sin(q)$
- ⁶ $-\frac{13}{85}$ or equivalent fraction
- ⁷ $-\frac{84}{13}$ or equivalent fraction (eg $-\frac{7140}{1105}$) 3 marks

Notes

1 •¹ and •² may, if necessary, be awarded as follows

$$\begin{aligned} \bullet^1 \quad \sin(p) &= \frac{15}{17}, \sin(q) = \frac{6}{10} \\ \bullet^2 \quad \cos(p) &= \frac{8}{17}, \cos(q) = \frac{8}{10} \end{aligned}$$

2 For •⁴

There has to be some working to show the completion.

$$\begin{aligned} \text{eg} \quad \dots &= \frac{120+48}{170} = \frac{168}{170} = \frac{84}{85} \\ \text{or} \quad \dots &= \frac{60}{85} + \frac{24}{85} = \frac{84}{85} \\ \text{or} \quad \dots &= \frac{12}{17} + \frac{24}{85} = \frac{84}{85} \end{aligned}$$

3 Calculating approx angles using $\text{inv} \sin$ and $\text{inv} \cos$ can gain no credit at any point.

4 Any attempt to use $\sin(p+q) = \sin(p) + \sin(q)$ **loses •3 and •4.**

Any attempt to use $\cos(p+q) = \cos(p) + \cos(q)$ **loses •5 and •6.**

This second option must not be treated as a repeated error.

Alternative 1 (for marks 3 & 4)

$$\begin{aligned} \bullet^3 \quad \frac{21}{\sin(p+q)} &= \frac{10}{\frac{8}{17}} \\ \bullet^4 \quad 10\sin(p+q) &= \frac{168}{17} \text{ and complete} \end{aligned}$$

Alternative 2 (for marks 5 & 6)

$$\begin{aligned} \bullet^5 \quad \cos(p+q) &= \frac{17^2 + 10^2 - 21^2}{2 \cdot 17 \cdot 10} \\ \bullet^6 \quad -\frac{13}{85} \end{aligned}$$

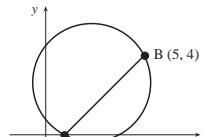
Alternative 3 (for marks 5 & 6)

$$\begin{aligned} \bullet^5 \quad \cos^2(p+q) &= 1 - \left(\frac{84}{85}\right)^2 \\ \bullet^6 \quad \cos(p+q) &= -\frac{13}{85} \text{ with justification} \\ &\text{of the choice of negative sign} \\ &\text{e.g. } (15+6)^2 (= 441) > 17^2 + 10^2 (= 389) \\ &\text{or using the cosine rule} \end{aligned}$$

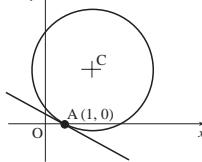
3 (a) A chord joins the points A(1, 0) and B(5, 4) on the circle as shown in the diagram.
Show that the equation of the perpendicular bisector of chord AB is $x + y = 5$.

(b) The point C is the centre of this circle. The tangent at the point A on the circle has equation $x + 3y = 1$.
Find the equation of the radius CA.

(c) (i) Determine the coordinates of the point C.
(ii) Find the equation of the circle.



4



4

4

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
3	a	4	C	G7	CN	05/44
	b	4	C	G15	CN	
	c	4	C	G10	CN	

The primary method m/s is based on the following generic m/s.
THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE
GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE
THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD
SHOWN IN DETAIL IN THE MARKING SCHEME

- ¹ ss: find perp. bisector
- ² pd: calc. perp. gradient
- ³ ss: find approp. mid-point
- ⁴ ic: complete proof
- ⁵ ss: compare with $y = mx + c$
- ⁶ ic: state gradient
- ⁷ ss: find gradient of radius
- ⁸ ic: state equation of line
- ⁹ ss: solve sim. equations
- ¹⁰ pd: solve sim. equations
- ¹¹ ic: state equation of circle
- ¹² pd: calculate radius

Primary Method : Give 1 mark for each •

- ¹ $m_{AB} = 1$
- ² $m_{\perp} = -1$
- ³ midpoint = (3, 2)
- ⁴ $y - 2 = -1(x - 3)$ and complete [Notes 1,2] 4 marks
- ⁵ $y = -\frac{1}{3}x + \dots$ stated/implied by •6
- ⁶ $m_{tgt} = -\frac{1}{3}$
- ⁷ $m_{rad} = 3$ stated/implied by •8
- ⁸ $y - 0 = 3(x - 1)$ [Note 3] 4 marks
- ⁹ use $x + y = 5$ and $y = 3x - 3$ [Notes 4,5]
- ¹⁰ $x = 2, y = 3$
- ¹¹ $(x - 2)^2 + (y - 3)^2 = r^2$
- ¹² $r^2 = 10$ [Note 6] 4 marks

Notes

1 To gain •4 some evidence of completion needs to be shown

eg
$$\begin{aligned} y - 2 &= -1(x - 3) \\ y - 2 &= -x + 3 \\ y + x &= 5 \end{aligned}$$

2 •4 is only available if an attempt has been made to find and use both a perpendicular gradient and a midpoint.

3 •8 is only available if an attempt has been made to find and use a perpendicular gradient.

4 At the •9, •10 stage

Guessing (2,3) (from stepping) and checking it lies on perp. bisector of AB may be awarded •9 and •10
Guessing (2,3) (with or without reason) and with no check gains **neither** •9 nor •10

5 Solving $y = 3x - 3$ and $x + 3y = 1$ leading to (1,0) will lose •9 and •10.

6 to gain •12 some evidence of use of the distance formula needs to be shown.

7 At the •11 and •12 stage

Subsequent to a guess for the coordinates of C, •11 and •12 are only available if the guess is such that $0 < x < 5$ and $0 < y < 4$.

Alternative 1 [for •9 and •10]

- ⁹ D=(3,6) where D is intersection of the perp. to AB through B and the circle.
- ¹⁰ C = midpoint of AD = (2,3)

Common Error 1 [for •5 to •8]

$$\begin{aligned} 3y &= -x + 1 \\ m &= -1 \\ m_{rad} &= 1 \\ y - 0 &= 1(x - 1) \\ \bullet 5 \times & \bullet 6 \times \bullet 7 \times \text{eased} \quad \bullet 8 \times \checkmark \\ \text{award 1 mark} \end{aligned}$$

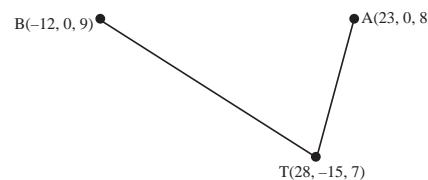
Common Error 2 [for •5 to •8]

$$\begin{aligned} x + 3y &= 1 \text{ so } m = 3 \\ y - 0 &= 3(x - 1) \\ \text{award 0 marks} \end{aligned}$$

4 The sketch shows the positions of Andrew(A), Bob (B) and Tracy(T) on three hill-tops.

Relative to a suitable origin, the coordinates (in hundreds of metres) of the three people are A(23, 0, 8), B(-12, 0, 9) and T(28, -15, 7).

In the dark, Andrew and Bob locate Tracy using heat-seeking beams.



(a) Express the vectors \vec{TA} and \vec{TB} in component form. 2

(b) Calculate the angle between these two beams. 5

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
4	a	2	C	G17	CN	05/55
	b	5	C	G28	Ca	

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GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE
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SHOWN IN DETAIL IN THE MARKING SCHEME

- ¹ ic: state vector components
- ² ic: state vector components
- ³ pd: find length of vector
- ⁴ pd: find length of vector
- ⁵ pd: find scalar product
- ⁶ ss: use scalar product
- ⁷ pd: evaluate angle

Notes

In (a)

- 1 For calculating \vec{AT} and \vec{BT} award 1 mark out of 2.
- 2 Treat column vectors written like $(-40, 15, 2)$ as bad form.
- In (b)
 - 3 For candidates who do not attempt •7, the formula quoted at •6 must relate to the labelling in the question for •6 to be awarded.
 - 4 Do not penalise premature rounding.

5 The use of $\tan(A\hat{T}B) = \frac{\vec{TA} \cdot \vec{TB}}{|\vec{TA}| |\vec{TB}|}$ loses •6

6 The use of $\cos(A\hat{T}B) = \frac{\vec{TA} \cdot \vec{TB}}{|\vec{TA}| |\vec{AB}|}$ means that only •5 and •7 are available.

Primary Method : Give 1 mark for each •

- ¹ $\vec{TA} = \begin{pmatrix} -5 \\ 15 \\ 1 \end{pmatrix}$
- ² $\vec{TB} = \begin{pmatrix} -40 \\ 15 \\ 2 \end{pmatrix}$ [Notes 1,2] 2 marks
- ³ $|\vec{TA}| = \sqrt{251}$
- ⁴ $|\vec{TB}| = \sqrt{1829}$
- ⁵ $\vec{TA} \cdot \vec{TB} = 427$
- ⁶ $\cos(A\hat{T}B) = \frac{\vec{TA} \cdot \vec{TB}}{|\vec{TA}| |\vec{TB}|}$ stated or implied by •7 [Note 3] 5 marks
- ⁷ $A\hat{T}B = 50.9^\circ$ OR 0.889° [Note 4]
OR 56.6 grads

Alternative 1 for •3 to •7 (Cosine Rule)

- ³ $|\vec{TA}| = \sqrt{251}$
- ⁴ $|\vec{TB}| = \sqrt{1829}$
- ⁵ $|\vec{AB}| = \sqrt{1226}$
- ⁶ $\cos(A\hat{T}B) = \frac{1829 + 251 - 1226}{2\sqrt{1829}\sqrt{251}}$ stated or implied by •7
- ⁷ $A\hat{T}B = 50.9^\circ$

Common Error No.1

$$\begin{aligned} \bullet^1 &\times \quad \vec{TA} = t - a = \begin{pmatrix} 5 \\ -15 \\ -1 \end{pmatrix} \\ \bullet^2 &\times \sqrt{} \quad \vec{TB} = t - b = \begin{pmatrix} 40 \\ -15 \\ -2 \end{pmatrix} \end{aligned}$$

award 1 mark

Common Error No.2

$$\begin{aligned} \bullet^1 &\times \quad \vec{TA} = t + a = \begin{pmatrix} 51 \\ -15 \\ 15 \end{pmatrix} \\ \bullet^2 &\times \sqrt{} \quad \vec{TB} = t + b = \begin{pmatrix} 16 \\ -15 \\ 16 \end{pmatrix} \end{aligned}$$

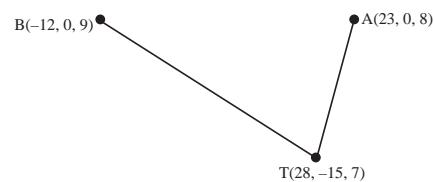
award 1 mark

Further common errors overleaf.

4 The sketch shows the positions of Andrew(A), Bob (B) and Tracy(T) on three hill-tops.

Relative to a suitable origin, the coordinates (in hundreds of metres) of the three people are A(23, 0, 8), B(-12, 0, 9) and T(28, -15, 7).

In the dark, Andrew and Bob locate Tracy using heat-seeking beams.



(a) Express the vectors \vec{TA} and \vec{TB} in component form. 2

(b) Calculate the angle between these two beams. 5

Common Error 1 : Finding angle BOA

$$\text{using } \vec{OB} = \begin{pmatrix} -12 \\ 0 \\ 9 \end{pmatrix} \text{ and } \vec{OA} = \begin{pmatrix} 23 \\ 0 \\ 8 \end{pmatrix}$$

- $|\vec{OB}| = \sqrt{225}$ and $|\vec{OA}| = \sqrt{593}$
- $\vec{OB} \cdot \vec{OA} = -204$
- $\cos(B\hat{O}A) = \frac{\vec{OB} \cdot \vec{OA}}{|\vec{OB}| |\vec{OA}|}$
- $B\hat{O}A = 124.0^\circ \text{ OR } 2163^\circ$

award 1 mark per bullet

Common Error 2 : Finding angle BOT

$$\text{using } \vec{OB} = \begin{pmatrix} -12 \\ 0 \\ 9 \end{pmatrix} \text{ and } \vec{OT} = \begin{pmatrix} 28 \\ -15 \\ 7 \end{pmatrix}$$

- $|\vec{OB}| = \sqrt{225}$ and $|\vec{OT}| = \sqrt{1058}$
- $\vec{OB} \cdot \vec{OT} = -273$
- $$\begin{cases} \cos(B\hat{O}T) = \frac{\vec{OB} \cdot \vec{OT}}{|\vec{OB}| |\vec{OT}|} \\ B\hat{O}T = 124.0^\circ \text{ OR } 2163^\circ \end{cases}$$

award 1 mark per bullet

Common Error 3 : Finding angle AOT

$$\text{using } \vec{OA} = \begin{pmatrix} 23 \\ 0 \\ 8 \end{pmatrix} \text{ and } \vec{OT} = \begin{pmatrix} 28 \\ -15 \\ 7 \end{pmatrix}$$

- $|\vec{OA}| = \sqrt{593}$ and $|\vec{OT}| = \sqrt{1058}$
- $\vec{OA} \cdot \vec{OT} = 700$
- $$\begin{cases} \cos(A\hat{O}T) = \frac{\vec{OA} \cdot \vec{OT}}{|\vec{OA}| |\vec{OT}|} \\ A\hat{O}T = 27.9^\circ \text{ OR } 0.487^\circ \end{cases}$$

award 1 mark per bullet

Common Error 4 : Finding angle ABT

$$\text{using } \vec{BA} = \begin{pmatrix} 35 \\ 0 \\ -1 \end{pmatrix} \text{ and } \vec{BT} = \begin{pmatrix} 40 \\ -15 \\ -2 \end{pmatrix}$$

- $|\vec{BA}| = \sqrt{1226}$ and $|\vec{BT}| = \sqrt{1829}$
- $\vec{BA} \cdot \vec{BT} = 1402$
- $$\begin{cases} \cos(A\hat{B}T) = \frac{\vec{BA} \cdot \vec{BT}}{|\vec{BA}| |\vec{BT}|} \\ A\hat{B}T = 20.6^\circ \text{ OR } 0.359^\circ \end{cases}$$

award 1 mark per bullet

Common Error 5 : Finding angle BAT

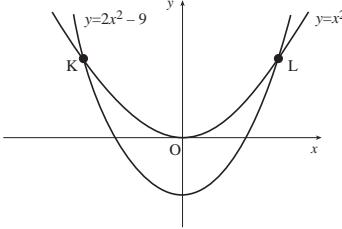
$$\text{using } \vec{AB} = \begin{pmatrix} -35 \\ 0 \\ 1 \end{pmatrix} \text{ and } \vec{AT} = \begin{pmatrix} 5 \\ -15 \\ -1 \end{pmatrix}$$

- $|\vec{AB}| = \sqrt{1226}$ and $|\vec{AT}| = \sqrt{251}$
- $\vec{AB} \cdot \vec{AT} = -176$
- $$\begin{cases} \cos(B\hat{A}T) = \frac{\vec{AB} \cdot \vec{AT}}{|\vec{AB}| |\vec{AT}|} \\ B\hat{A}T = 108.5^\circ \text{ OR } 1.894^\circ \end{cases}$$

award 1 mark per bullet

5 The curves with equations $y = x^2$ and $y = 2x^2 - 9$ intersect at K and L as shown.

Calculate the area enclosed between the curves.



8

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
5		8	C	C17	CN	05/49

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- ¹ ss: find intersection
- ² pd: process quadratic equ.
- ³ ss: upper – lower
- ⁴ ic: interpret limits
- ⁵ pd: sub. & simplify Upper – Lower
- ⁶ pd: integrate
- ⁷ ic: substitute limits
- ⁸ pd: evaluate and complete

Primary Method : Give 1 mark for each •

- ¹ $x^2 = 2x^2 - 9$
- ² $x = \pm 3$
- ³ \int_{upper}^{lower} [Notes 3,4] stated or implied by •5
- ⁴ eg $\int_0^3 \dots$
- ⁵ $x^2 - 2x^2 + 9$
- ⁶ $\left[-\frac{1}{3}x^3 + 9x \right]_0^3$
- ⁷ $\left(-\frac{1}{3} \times 3^3 + 9 \times 3 \right) - 0$
- ⁸ $2 \times 18 = 36$ [Note 3] 8 marks

Notes

- 1 There is no penalty for working with $\frac{1}{3}x^3 - \frac{2}{3}x^3 + 9x$ or even $\frac{1}{3}x^3 - \left(\frac{2}{3}x^3 - 9x\right)$ but in the latter case, the minus signs need to be dealt with correctly at some point for •5 to be awarded.
- 2 Candidates who attempt to find a solution using a graphics calculator earn no marks. The only acceptable solution is via calculus.
- 3 •3 is lost for subtracting the wrong way round and subsequently •8 may be lost for such statements as

–36
–36 square units
–36 = 36
–36 so ignore the –ve
–36 = 36 square units

•8 may be gained for statements such as

–36 so the area = 36

- 4 $\int_{-3}^3 (lower - upper)$ or $\int_{-3}^0 (lower - upper)$

are technically correct and hence all 8 marks are available.

- 5 For $\int_K^L (upper - lower)$, •3, •5, •6 and •7 are available

- 6 Differentiation loses •6, •7 and •8.

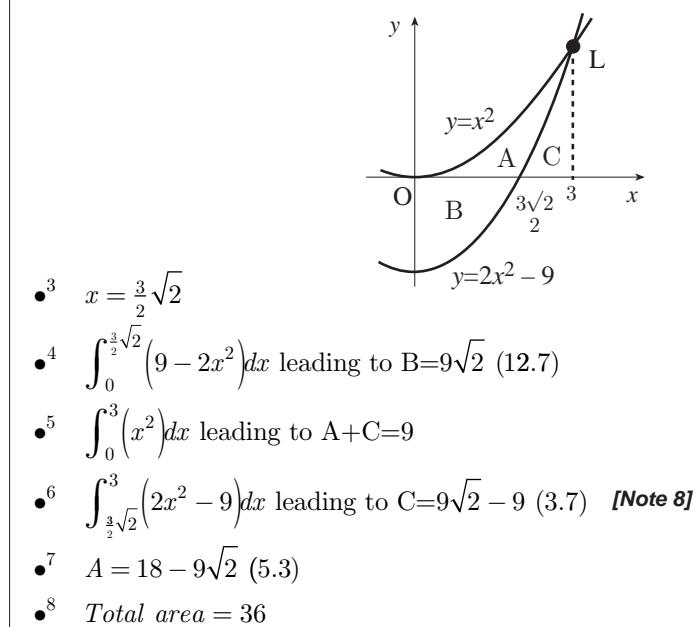
- 7 Using $x^2 + 2x^2 - 9$ and $\int_{-3}^3 (3x^2 - 9)dx$ leading to zero can only gain •4 and •6 from the last 6 marks.

- 8 Candidates may attempt to split the area up. In Alt.2, for candidates who treat "C" as a triangle, the last three marks are not available.

Alternative 1 for •4 to •8

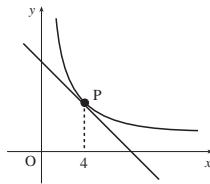
- ⁴ eg $\int_{-3}^3 \dots$
- ⁵ $x^2 - 2x^2 + 9$
- ⁶ $\left[-\frac{1}{3}x^3 + 9x \right]_{-3}^3$
- ⁷ $\left(-\frac{1}{3} \times 3^3 + 9 \times 3 \right) - \left(-\frac{1}{3} \times (-3)^3 + 9 \times (-3) \right)$
- ⁸ 36

Alternative 2 for •3 to •8



10

6 The diagram shows the graph of $y = \frac{24}{\sqrt{x}}$, $x > 0$.



Find the equation of the tangent at P, where $x = 4$.

6

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
6		6	B	C5, C3	CN	05/43

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- ¹ ss : know to differentiate
- ² ic : express in st. form
- ³ pd : differentiate -ve fractional index
- ⁴ pd : evaluate -ve fractional index
- ⁵ pd : evaluate y-coord
- ⁶ ic : state equ of tangent

Primary Method : Give 1 mark for each •

- ¹ $\frac{dy}{dx} = \dots$
- ² $y = 24x^{-\frac{1}{2}}$
- ³ $\frac{dy}{dx} = -12x^{-\frac{3}{2}}$
- ⁴ $\frac{dy}{dx}_{x=4} = -\frac{3}{2}$
- ⁵ $y_{x=4} = 12$
- ⁶ $y - 12 = -\frac{3}{2}(x - 4)$ [Notes 1,2,3] **6 marks**

nr $[2y + 3x = 36]$

nr = not required

Notes

- 1 •⁴ and •⁶ are only available if an attempt to find the gradient is based on differential calculus.
- 2 •⁶ is not available to candidates who find and use a perpendicular gradient.
- 3 •⁶ is only available for a numerical value of m .

Common Error 1

- ¹ $\frac{dy}{dx} = \dots$
- ² $y = 24x^{-\frac{1}{2}}$
- ³ $\frac{dy}{dx} = \frac{24x^{\frac{1}{2}}}{\frac{1}{2}}$
- ⁴ $\frac{dy}{dx}_{x=4} = 96$
- ⁵ $y_{x=4} = 12$
- ⁶ $y - 12 = 96(x - 4)$

- ¹ ✓
- ² ✓
- ³ ✗
- ⁴ ✗ eased
- ⁵ ✓
- ⁶ ✗✓

award 4 marks

Common Error 2

- ¹
- ² $y = 24x^{-\frac{1}{2}}$
- ³ $\int 24x^{-\frac{1}{2}}dx = \frac{24x^{\frac{1}{2}}}{\frac{1}{2}} + c$
- ⁴ gradient = 96
- ⁵ $y_{x=4} = 12$
- ⁶ $y - 12 = 96(x - 4)$

- ¹ ✗
- ² ✓
- ³ ✗
- ⁴ ✗ Note 1
- ⁵ ✓
- ⁶ ✗ Note 1

award 2 marks

7 Solve the equation $\log_4(5-x) - \log_4(3-x) = 2$, $x < 3$.

4

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
7		4	A	A7	CN	0525

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- ¹ ss: use the log laws
- ² ss: know to convert from log to expo
- ³ pd: process conversion
- ⁴ pd: find valid solution

Primary Method : Give 1 mark for each •

- ¹ $\log_4\left(\frac{5-x}{3-x}\right)$
- ² use $\log_a(b) = c \Leftrightarrow b = a^c$ stated or implied by •3
- ³ $\frac{5-x}{3-x} = 4^2$ See Cave
- ⁴ $x = \frac{43}{15}$ 4 marks

Notes

1 For •4

Accept answer as a decimal.

Common Error No.1

- 1 ✓ $\log_4\left(\frac{5-x}{3-x}\right) = \log_4(8)$
- 2 ✗
- 3 ✗
- 4 ✗✓ $\frac{5-x}{3-x} = 8$
- 4 ✗✓ $x = \frac{19}{7}$

*award 2 marks***Common Error No.2**

- 1 ✓ $\log_4\left(\frac{5-x}{3-x}\right) = 2$
- 2 ✗ $\frac{5-x}{3-x} = 2$
- 3 ✗
- 4 ✗✓ $x = \frac{1}{2}$
- 4 ✗✓ $x = 7$ which is not a valid sol.

*award 2 marks***Common Error No.3**

- 1 ✓ $\log_4\left(\frac{5-x}{3-x}\right) = 2$
- 2 ✗ $\log_4\left(\frac{5-x}{3-x}\right) = \log_4 2$
- 3 ✗ $\frac{5-x}{3-x} = 2$
- 4 ✗✓ $x = 1$.

*award 2 marks***Alternative 1**

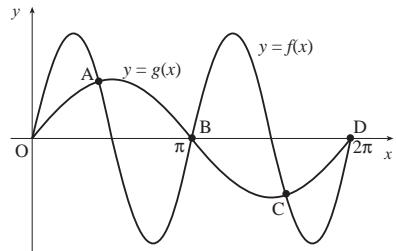
- ¹ $\log_4\left(\frac{5-x}{3-x}\right)$
- ² $2\log_4 4$ stated or implied by •3
- ³ $\left(\frac{5-x}{3-x}\right) = 4^2$
- ⁴ $x = \frac{43}{15}$

Cave

$\log_4\left(\frac{5-x}{3-x}\right)$	BUT	$\log_4\left(\frac{5-x}{3-x}\right)$
$\frac{5-x}{3-x} = 16$		$\frac{5-x}{3-x} = 2^4$
leading to $x = \frac{43}{15}$		leading to $x = \frac{43}{15}$

award 4 marks *•¹ ✓, •² ✗, •³ ✗, •⁴ ✗✓*
award 2 marks

8 Two functions, f and g , are defined by
 $f(x) = k \sin(2x)$ and $g(x) = \sin(x)$ where $k > 1$.
The diagram shows the graphs of
 $y = f(x)$ and $y = g(x)$ intersecting at O, A, B, C and D. Show that, at A and C, $\cos(x) = \frac{1}{2k}$.



5

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
8		5	A	T10	CN	05/47

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SHOWN IN DETAIL IN THE MARKING SCHEME

- ¹ ss: equate for intersection
- ² ss: use double angle formula
- ³ pd: factorise
- ⁴ pd: process two solutions
- ⁵ ic: complete proof

Primary Method : Give 1 mark for each •

- ¹ $k \sin(2x) = \sin(x)$ [Note 1]
- ² $k \times 2 \sin(x) \cos(x)$
- ³ $\sin(x)(2k \cos(x) - 1) = 0$
- ⁴ $\sin(x) = 0$
 $and \cos(x) = \frac{1}{2k}$
- ⁵ $\sin(x) = 0 \Rightarrow x = 0, \pi, 2\pi$
i.e. at (O), B and D [Note 2]

and $\cos(x) = \frac{1}{2k}$ is for A and C.

5 marks

Notes

- 1 Only •1 is available for candidates who substitute a numerical value for k at the start.
- 2 •5 is only available if a suitable comment regarding points (O), B and D is made.
- 3 If all the terms are transposed to one side, then an “=0” needs to appear at least once.
- 4 For Alternative 3
•4 and •5 are not available unless •3 has been awarded.

Alternative 1 for •4 and •5

- ⁴ at (O), B and D, $\sin(x) = 0$
- ⁵ so at A and C, $2k \cos(x) - 1 = 0$
 $\Rightarrow \cos(x) = \frac{1}{2k}$.

Alternative 2 for •4 and •5

- ⁴ at A and C, $\sin(x) \neq 0$
- ⁵ so at A and C, $2k \cos(x) - 1 = 0$
 $\Rightarrow \cos(x) = \frac{1}{2k}$.

Common Error 1

- ¹ ✓ $k \sin(2x) = \sin(x)$
- ² ✓ $k \times 2 \sin(x) \cos(x) - \sin(x) = 0$
- ³ ✓ $\sin(x)(2k \cos(x) - 1)$
- ⁴ ✗ $2k \cos(x) - 1 = 0$
- ⁵ ✗ $\cos(x) = \frac{1}{2k}$ at A and C.

award 3 marks

Common Error 2

- ¹ ✓ $k \sin(2x) = \sin(x)$
- ² ✓ $k \times 2 \sin(x) \cos(x) = \sin(x)$
- ³ ✗ $k \times 2 \cos(x) = 1$
- ⁴ ✗
- ⁵ ✗ $\cos(x) = \frac{1}{2k}$ at A and C.

award 2 marks

Alternative 3 for •1 to •5

- ¹ $k \sin(2x) = \sin(x)$
- ² $k \times 2 \sin(x) \cos(x) = \sin(x)$
- ³ at A and C, $\sin(x) \neq 0$
- ⁴ so at A and C, $2k \cos(x) = 1$
- ⁵ $\cos(x) = \frac{1}{2k}$

9 The value V (in £ million) of a cruise ship t years after launch is given by the formula

$$V = 252e^{-0.06335t}.$$

(a) What was its value when launched? 1
 (b) The owners decide to sell the ship once its value falls below £20 million. After how many years will it be sold? 4

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
9	a	1	B	A34	CN	05/76
	b	4	A	A34	Ca	

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- ¹ pd: evaluate at $t = 0$
- ² ic: substitute $V = 20$
- ³ pd: process
- ⁴ ic: expo to log conversion
- ⁵ pd: solve a logarithmic equation

Primary Method : Give 1 mark for each •

- ¹ $V_{t=0} = 252$ (£m) 1 mark
- ² $252e^{-0.06335t} = 20$
- ³ $e^{-0.06335t} = \frac{20}{252}$
- ⁴ $-0.06335t = \ln\left(\frac{20}{252}\right)$
- ⁵ $t = 40$ [Note 1] 4 marks

Notes

in (b)

- 1 For •5 accept any correct answer which rounds to 40.
- 2 An answer obtained by trial and improvement which rounds to 40 may be awarded a max. of 1 mark (out of 4) **but only** if they have checked 39 as well.
- 3 In following through from an error, •5 is only available for a positive answer.

Common Error 1

- ² ✓ $\log(252e^{-0.06335t}) = \log 20$
- ³ ✗ $-0.06335t \log 252 = \log 20$
- ⁴ ✗ $-0.06335t = \frac{\log 20}{\log 252}$
- ⁵ ✗ $t = -8.55$

award 1 mark

Alternative 1 for (b) (takings logs of both sides)

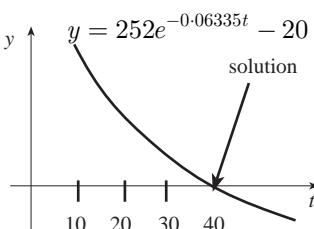
- ² $252e^{-0.06335t} = 20$
- ³ $e^{-0.06335t} = \frac{20}{252}$
- ⁴ $-0.06335t \log_k(e) = \log_k\left(\frac{20}{252}\right)$
where $k = e$ or $k = 10$
- ⁵ $t = 40$ [Note 1]

Alternative 2

- ² $252e^{-0.06335t} = 20$
- ³ $\log 252 - 0.06335t \log e = \log 20$
- ⁴ $5.53 - 0.06335t = 2.99$
- ⁵ $t = 40$

Solution via graphics calculator

- ² $252e^{-0.06335t} = 20$
- ³ choose to graph $y = 252e^{-0.06335t} - 20$
- ⁴ a sketch [see below]
- ⁵ $t = 40$



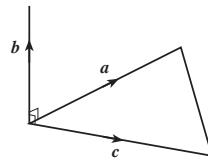
Alternative 3

- ² $252e^{-0.06335t} = 20$
- ³ $\ln 252 + \ln e^{-0.06335t} = \ln 20$
- ⁴ $-0.06335t \ln e = \ln 20 - \ln 252$
- ⁵ $t = 40$

Note

You could also graph, for example, $y = 252e^{-0.06335t}$ and $y = 20$

10 Vectors \mathbf{a} and \mathbf{c} are represented by two sides of an equilateral triangle with sides of length 3 units, as shown in the diagram. Vector \mathbf{b} is 2 units long and \mathbf{b} is perpendicular to both \mathbf{a} and \mathbf{c} . Evaluate the scalar product $\mathbf{a} \cdot (\mathbf{a} + \mathbf{b} + \mathbf{c})$.



4

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
10		4	A	G29	CN	05/31

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- ¹ ss: use distributive law
- ² pd: process scalar product
- ³ pd: process scalar product
- ⁴ pd: process scalar product & complete

Primary Method : Give 1 mark for each •

- ¹ $\mathbf{a} \cdot \mathbf{a} + \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$ see CAVE [Notes 1,2]
- ² $\mathbf{a} \cdot \mathbf{a} = 9$
- ³ $\mathbf{a} \cdot \mathbf{c} = \frac{9}{2}$ [Note 3]
- ⁴ $\mathbf{a} \cdot \mathbf{b} = 0$ and a total of $13\frac{1}{2}$

4 marks

Notes

- 1 Treat $\underline{\mathbf{a}} \cdot \mathbf{a}$ written as \mathbf{a}^2 as bad form.
- 2 Treat $\underline{\mathbf{a}} \cdot \mathbf{b}$ written as \mathbf{ab} as an error unless it is subsequently evaluated as a scalar product. Similarly for $\underline{\mathbf{a}} \cdot \mathbf{c}$.
- 3 Using $\underline{\mathbf{p}} \cdot \underline{\mathbf{q}} = |\mathbf{p}||\mathbf{q}|\sin\theta$ consistently loses 1 mark. (ie max. available is 3)
- 4 When attaching the components

$$\mathbf{c} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}, \mathbf{a} = \begin{pmatrix} \frac{3}{2} \\ \frac{3\sqrt{3}}{2} \\ 0 \end{pmatrix}, \text{ all marks are available.}$$

When attaching the components

$$\mathbf{c} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}, \mathbf{a} = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}, \text{ only } \bullet 1 \text{ is available.}$$

CAVE

$\mathbf{a} \cdot (\mathbf{a} + \mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{a} + \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$
followed by
 $\mathbf{a} \cdot \mathbf{a} = 9$
earns $\bullet 1$ and $\bullet 2$.

but
 $\mathbf{a} \cdot (\mathbf{a} + \mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{a} + \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$
followed by
 $\mathbf{a} \cdot \mathbf{a} = 9, \mathbf{a} \cdot \mathbf{c} = 9, \mathbf{a} \cdot \mathbf{b} = 6$
earns $\bullet 1$ only.

11 (a) Show that $x = -1$ is a solution of the cubic equation $x^3 + px^2 + px + 1 = 0$. 1
 (b) Hence find the range of values of p for which all the roots of the cubic equation are real. 7

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
11	a	1	C	A21	CN	05/54
	b	7	A	A22	CN	

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- ¹ pd: evaluate the function at $x = -1$
- ² ss: strategy for finding other factors
- ³ ic: quadratic factor
- ⁴ ss: strategy for real roots
- ⁵ ic: substitute
- ⁶ pd: process
- ⁷ ss: starts to solve inequation
- ⁸ ic: complete

Primary Method : Give 1 mark for each •

- ¹ $f(-1) = -1 + p - p + 1 = 0$ 1 mark
- ²
$$\begin{array}{c|ccccc} -1 & 1 & p & p & 1 \\ & -1 & 1-p & -1 \\ \hline & 1 & p-1 & 1 & 0 \end{array}$$
- ³ $x^2 + (p-1)x + 1 = 0$ [Note 2]
- ⁴ " $b^2 - 4ac$ " and " ≥ 0 " [Notes 3,4]
- ⁵ $(p-1)^2 - 4$
- ⁶ $(p-3)(p+1)$
- ⁷ $p = 3, p = -1$
- ⁸ $p \leq -1, p \geq 3$ [Note 6] 7 marks

Notes

- 1 For alternative method 1, •²
- 2 (as is •³ also) is for interpreting the result of a synthetic division.
 Candidates must show some acknowledgement of the result of the synthetic division. Although a statement w.r.t. the zero is preferable, accept something as simple as "underlining" the zero.
- 2 Treat " $= 0$ " missing at •³ as Bad Form
- 3 •⁴ is only available as a consequence of obtaining a quadratic factor from a division of the cubic.
- 4 Using $b^2 - 4ac > 0$ loses •⁴
 An " \geq " must appear at least once somewhere between •⁴ and •⁶
- 5 Where errors occur at the •³•⁵ stages, then •⁶, •⁷, •⁸ are still available for solving a '3-term' quadratic inequation.
- 6 Evidence for •⁸ may be a table of values or a sketch

7 For candidates who start with $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, all marks

are available (subject to working being equivalent to the Primary Method).

8 Wrong discriminant:

Using $b^2 + 4ac$ only •⁵ (out of the last 5 marks) is available.

Any other expression masquerading as the discriminant loses all of the last 5 marks.

Alternative method 1 for marks 1,2 (starting with synth. division)

$$\bullet^1 \begin{array}{c|ccccc} -1 & 1 & p & p & 1 \\ & -1 & 1-p & -1 \\ \hline & 1 & p-1 & 1 & 0 \end{array}$$

$$\bullet^2 f(-1) = 0 \quad \text{[Note 1]}$$

etc

Marks should still be recorded as out of 1 and 7

Alternative method 2 for marks 1,2 (quad. factor obtained by inspection)

$$\bullet^1 f(-1) = -1 + p - p + 1 = 0$$

$$\bullet^2 f(x) = (x+1)(x^2 \dots)$$

etc

Common Error 1 (marks 5 to 8)

$$(p-1)^2 - 4 \geq 0$$

$$(p-1)^2 \geq 4$$

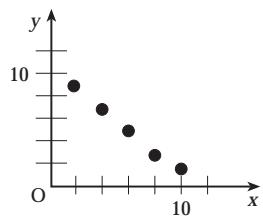
$$p-1 \geq 2$$

$$p \geq 3$$

award 2 marks out of last 4

[4] The scatter diagram shows 5 pairs of data values for x and y where $\Sigma x = 30$, $\Sigma y = 26$, $\Sigma x^2 = 220$, $\Sigma y^2 = 168$ and $\Sigma xy = 120$.

(a) Find the equation of the regression line.
 (b) Estimate the value of y when $x = 5$.



4
1

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
S1	a	4	C	4.4.2	Ca	05/76
	b	1	C	4.4.2	CN	

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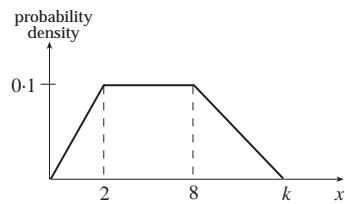
- ¹ pd: calculate S_{xy}
- ² pd: calculate S_{xx}
- ³ pd: calculate b
- ⁴ pd: calculate a & state equ.
- ⁵ ic: use equ. of regression line

Primary Method : Give 1 mark for each •

- ¹ $S_{xy} = -36$
- ² $S_{xx} = 40$
- ³ $b = -0.9$
- ⁴ $a = 10.6$ and $y = 10.6 - 0.9x$ 4 marks
- ⁵ $y_{x=5} = 6.1$ 1 mark

[7] The diagram represents the probability density function for a continuous random variable X .

(a) Find the value of k .
 (b) Find the median.



3
2

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
S4	a	3	A	4.3.1	CN	05/83
	b	2	A	4.3.5	CN	

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- ¹ ss: state total area = 1
- ² ic: find expression for total area
- ³ pd: process
- ⁴ ss: know total area = 0.5
- ⁵ pd: process

Primary Method : Give 1 mark for each •

- ¹ area = 1
- ² $0.1 + 0.6 + \frac{1}{2}(k - 8) \times 0.1$ 3 marks
- ³ $k = 14$
- ⁴ $0.1 + (m - 2) \times 0.1 = \frac{1}{2}$
- ⁵ $m = 6$ 2 marks

[9] (a) Explain briefly the difference between sample standard deviation and range as measures of spread.

(b) In statistics mode, a calculator shows the summary statistics for a certain data set. One data value, 1.2, is shown to be erroneous and is deleted. Calculate the sample standard deviation of the new data set of 19 values correct to 3 decimal places.

$\bar{x} =$	2.325
$S_x =$	0.573883355
$\sigma_x =$	0.559352304
$\Sigma x =$	46.5
$\Sigma x^2 =$	114.37
$n =$	20
$x_{\min} =$	1.2
$x_{\max} =$	3.2

1
4

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
S3	a	1	B	4.2.11/12	CN	05/79
	b	4	B	4.1.1	Ca	

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- ¹ ic: explanation
- ² pd: find new $\sum x$
- ³ pd: find new $\sum x^2$
- ⁴ ss: use formula for S_x
- ⁵ pd: process

Primary Method : Give 1 mark for each •

- ¹ SD is a measure of spread about mean whereas $(x_{\max} - x_{\min})$ is a measure of range. **1 mark**
- ² $\Sigma x = 45.3$
- ³ $\Sigma x^2 = 112.93$
- ⁴ $S = \sqrt{\frac{1}{18} \left(112.93 - \frac{45.3^2}{19} \right)}$
- ⁵ 0.523 **4 marks**

[10] A large organisation decides to run a mini-lottery for charity.

- Each participant selects any three different numbers from 1 to 20 inclusive.
- Every Friday the three winning numbers are drawn at random from the 20.
- Each participant with these winning numbers shares the jackpot.

(a) Find the number of possible combinations and hence find the probability of a particular combination winning a share of the jackpot.

(b) Find the probability that someone chooses the winning combination exactly twice within 3 successive weeks.

2
3

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
S4	a	2	B	4.2.5, 4.2.3	Ca	05/78
	b	3	A	4.2.7	Ca	

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- ¹ ss: find combination
- ² pd: calculate probability
- ³ ic: interpret $p(\text{win})$
- ⁴ ss: find combination
- ⁵ pd: process

Primary Method : Give 1 mark for each •

- ¹ No. of outcomes = $\binom{20}{3}$
- ² $prob = \frac{1}{\binom{20}{3}} = \frac{1}{1140}$ **2 marks**
- ³ $p(L) = \frac{1139}{1140}$
- ⁴ $p(2 \text{ wins in } 3)$
 $= 3 \times \left(\frac{1}{1140}\right)^2 \times \left(\frac{1139}{1140}\right)$
- ⁵ 2.306×10^{-6} **3 marks**

18